

Analysis of neutron and photon data with unresolved peak density

(work in progress and thus preliminary)

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In heavy nuclei the increase of the level density with excitation energy usually results in not fully resolved spectra: the experimental resolution exceeds the average level distance and more and more spectral lines or resonances escape a proper detection. In most cases it is justified to use results obtained for Gaussian ensembles by random matrix theory to derive correction procedures for the missing information. Distributions of neutron, radiation and fission widths observed in neutron capture experiments were analyzed by Porter and Thomas in terms of chi-square distributions already 50 years ago. Since then the specific feature of such distributions – the co-occurrence of single strong peaks together with copious very weak lines – have been observed in many other nuclear processes and occasionally the missing regard of this feature has resulted in an improper interpretation of respective data.

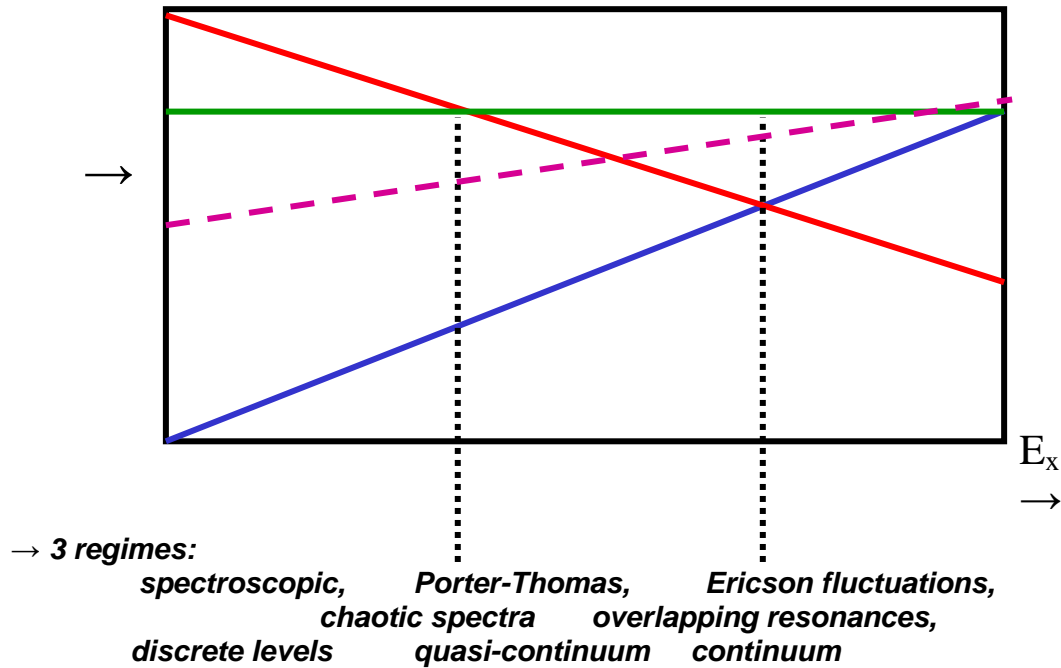
It seems worthwhile to apply improved analysis methods to examine current high resolution data with respect to their statistical properties. This will eventually allow a better insight in gross structures of importance for predictions of reaction cross sections outside of regions investigated experimentally – like those reached with fast neutrons. Porter Thomas play an important role not only in nuclear physics, they appear when the typical range of configuration mixing exceeds the average level distance. In data with high experimental resolution the resulting effects can be surprisingly large.

New data – after a proper investigation of their modification by Porter-Thomas effects – show discontinuities in the energy dependence of level distances, which should be further investigated experimentally as such discontinuities may be of importance for the proper understanding of many nuclear data. Very often a continuous increase of the level distance with excitation energy is taken for granted.

1. Gaussian ensembles and Porter-Thomas fluctuations

Nuclei at high excitation energy E_x show (in collective and single-particle dof)

1. Increasing level density $\rho \rightarrow$ decreasing average level distance D
2. Increasing average level width $\langle \Gamma \rangle \rightarrow$ decreasing life times τ



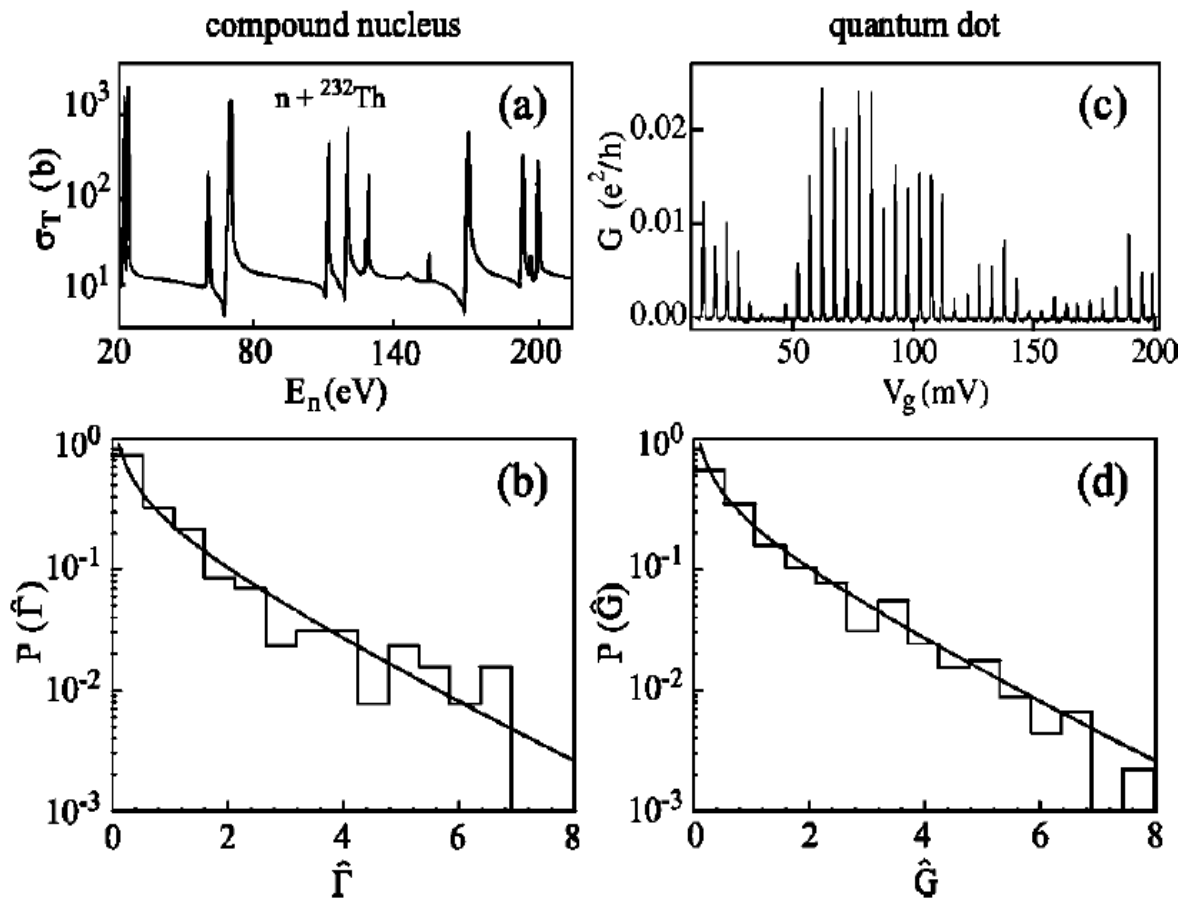
experimental resolution $\sigma = \sigma_{rms}$ plays important role for observations:

$$\Gamma \ll \sigma \leq D \quad \Gamma < D < \sigma \quad D < \sigma < \Gamma$$

Configuration mixing matrix elements M determine the purity of nuclear wave functions, they mix strongly for $D \leq M$

Statistical (random matrix) theory of quantum dots

Y. Alhassid, Rev. Mod. Phys. 72 (2000)



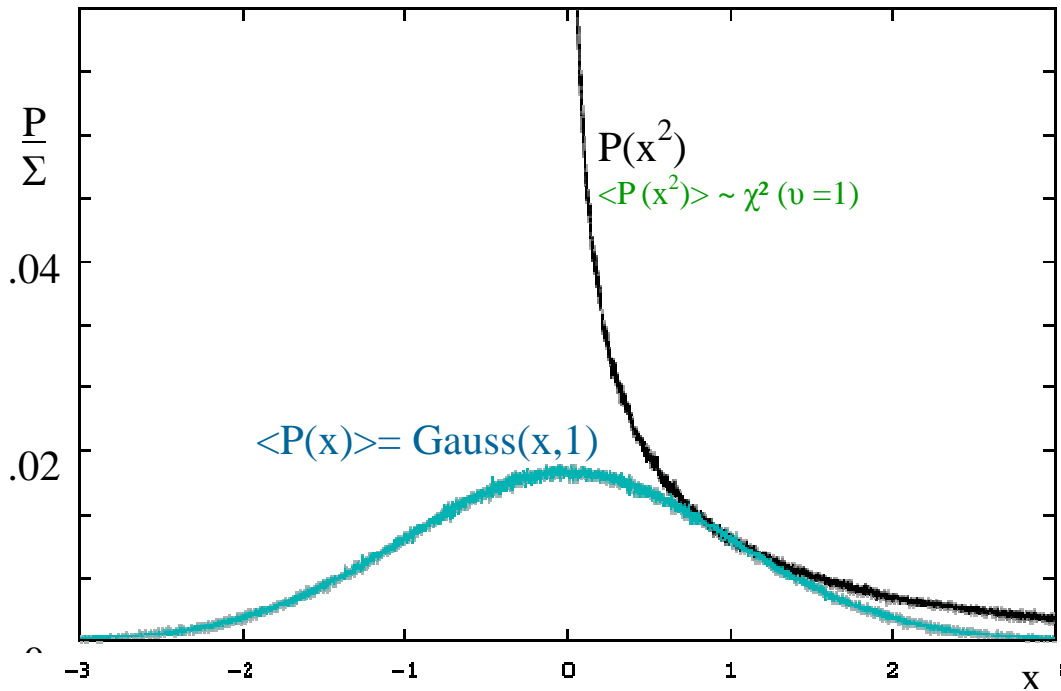
Neutron-resonance-width statistics in the compound nucleus

Coulomb-blockade peak statistics in closed quantum dots

- (a) neutron resonances in the total cross section of $n+{}^{232}\text{Th}$ as a function of the incoming neutron energy (BNL, 1964)
- (b) normalized neutron resonance widths in ${}^{233}\text{Th}$ vs the Porter-Thomas distribution predicted by Random Matrix Theory.
- (c) Coulomb-blockade peaks observed in the conductance of closed GaAs/AlGaAs dots as a function of gate voltage
- (d) distribution of the normalized conductance peak heights

The solid line is the prediction based on RMT and contains no free parameters.
 Notice the agreement over almost 3 orders of magnitude.

Porter-Thomas distribution of widths results from
Gaussian centered at 0 for the matrix elements



Strong PT-fluctuations
have been observed in:
nuclear reactions into
continuum region
 β -delayed particle-emission
e-scattering in the GDR-regime
n-absorption excitation functions
n-capture gamma-ray spectra
 γ -scattering in GDR tail (nrf)

– often together with real high energy structures
like:
giant resonances (GDR,..)
isobaric analog states
Gamov-Teller resonances
high spin yrast levels
'pygmy' structures

Besides **Porter-Thomas fluctuations**, characteristic for $\Gamma \ll D \approx \sigma$,
Ericsson fluctuations govern emission spectra, if $D \ll \Gamma \leq \sigma$.
They are typical for nuclear reaction spectra far above particle thresholds.
In heavy nuclei $D \ll \Gamma$ only occurs at $E_x > S_{n+} + 5-10$ MeV i.e. in and above GDR.

Near particle thresholds the
line width Γ can be smaller than or equal to the experimental resolution σ .

2. Statistical analysis of p-, n- and e-data (literature survey)

P. G. Hansen, Ann. Rev. Nucl. Part. Sci. 29 (1979) 69:

Exotic nuclei studies in on-line mass separators by β -delayed particle emission and fluctuation analysis of spectra

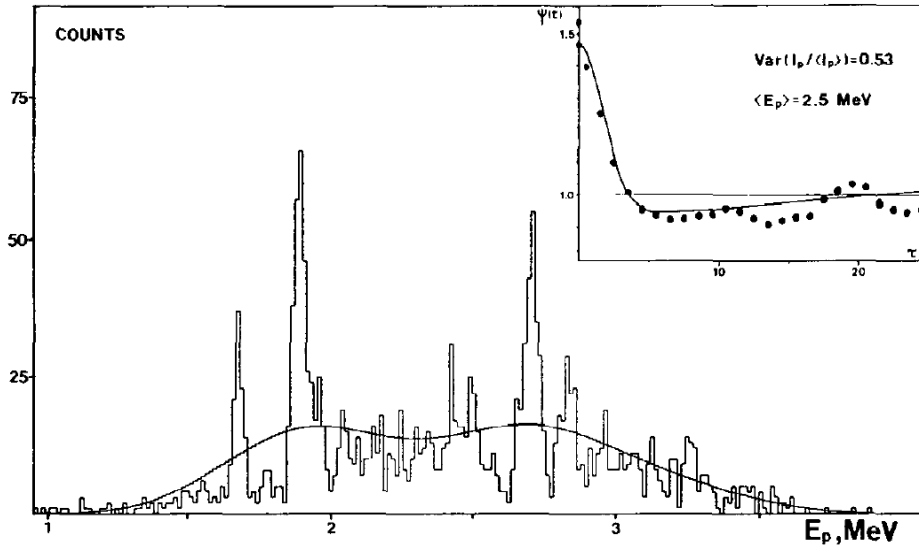
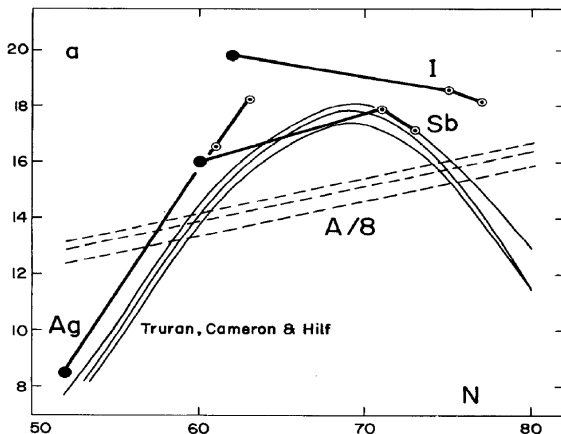


Figure 16 The measured proton energy spectrum of 16-s ^{99}Cd and a smoothed spectrum obtained with a folding function of second order. The inset shows the autocorrelation function $\psi(\tau)$ for a spectrum formed as the quotient of the two spectra. The solid curve shows a theoretical fit to the autocorrelation function, from which a level density parameter $a = 8.5$ was obtained (Elmroth et al 1978).

$$c(\epsilon) = 1 + \frac{\alpha D}{2\pi^{1/2}\sigma} \exp\left(-\frac{\tau^2}{4\sigma^2}\right)$$

$\nearrow 2\pi^{1/2} \cdot (c(0) - 1) \cdot \sigma = \alpha \cdot D$

$\alpha = 5, E_x \approx 5 \text{ MeV}, D \approx 16 \text{ keV}$



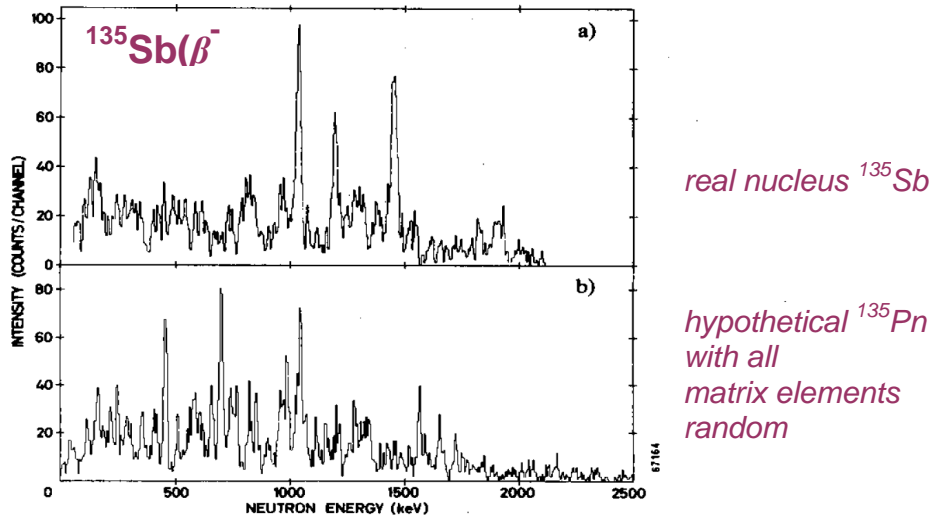
$c(\epsilon)$ is the autocorrelation function

$$\frac{\langle c(E+\epsilon) \cdot c(E) \rangle}{\langle c(E) \rangle^2}$$

and α a normalized variance;
it depends on the statistics of the process

The fluctuation analysis \bullet is reported to be in accordance to neutron resonance data \circ and the semiempirical formula of

Truran et al.



The energy spectrum of beta-delayed neutrons from ^{135}Sb from experiment (a) and from a Monte Carlo simulation of the fictional nucleus ^{135}Pn (b). The simulation used slowly varying strength functions corresponding to the parameters of the real decay. Thus the appreciable fine structure in (b) is essentially a result of Porter–Thomas fluctuations. (From J.C. Hardy, B. Jonson and P.G. Hansen, Nucl. Phys. **A305** (1978) 15)

Porter-Thomas distribution and fluctuations

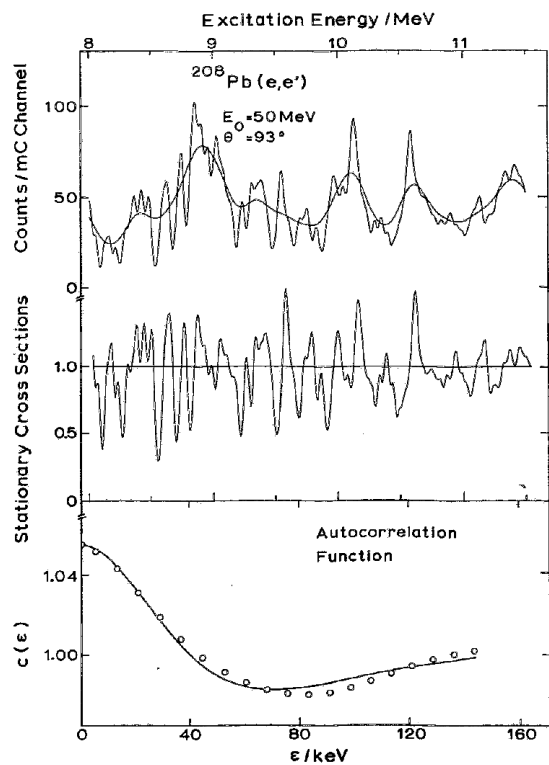
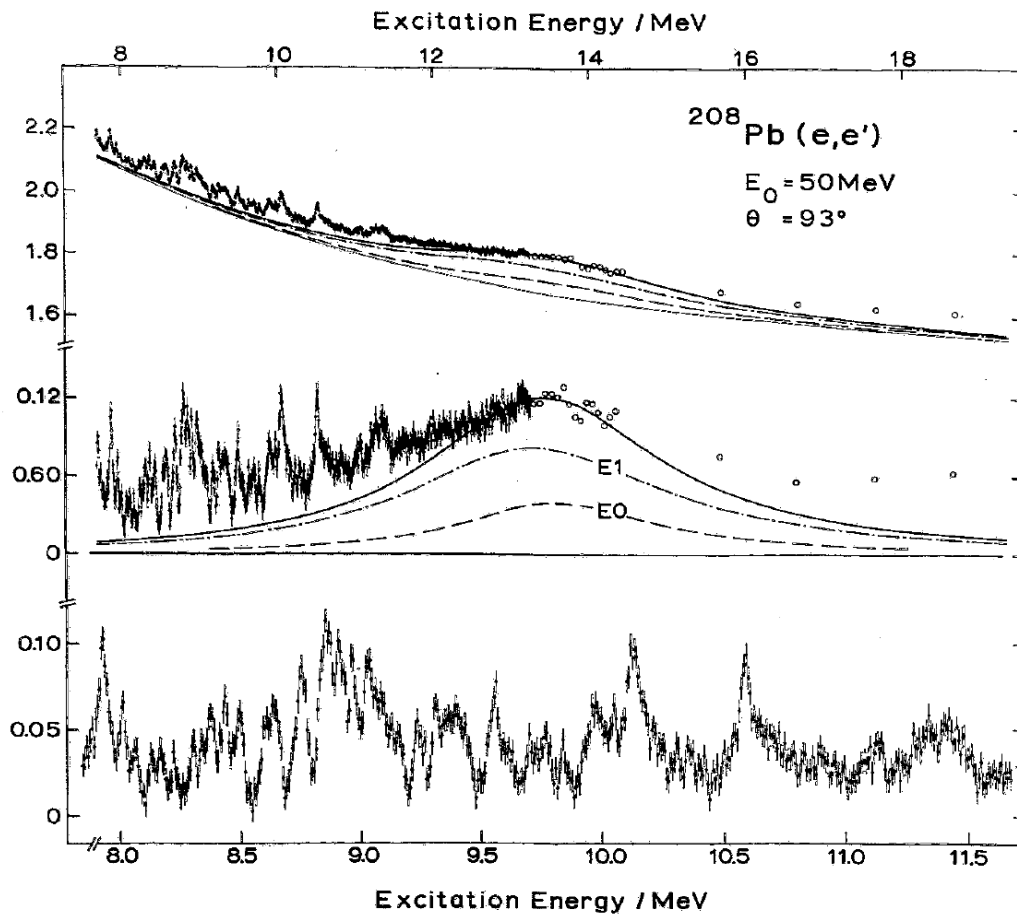
In the case of strong mixing of nuclear configurations reaction cross sections and gamma widths follow a Porter-Thomas distribution.

It is characterized by ample low intensity components and surprisingly strong outliers, simulating strong lines.

Large intensity fluctuations occur, if $D \ll \sigma$,
i.e. average level distance is much smaller than experimental resolution.

In respective experiments the observed intensity results from the folding of the underlying strength and the level density.

To untangle this, Porter Thomas fluctuations have to be accounted for properly.



The experimental autocorrelation function (5) can be well approximated [19] through the expression

$$c(\epsilon) = 1 + (\alpha \langle D \rangle / 2 \sqrt{\pi \sigma}) \{ \exp(-\epsilon^2 / 4 \sigma^2) + y^{-1} \exp(-\epsilon^2 / 4 \sigma^2 y^2) - \sqrt{8 / (1 + y^2)} \exp[-\epsilon^2 / 2 \sigma^2 (1 + y^2)] \} \quad (6)$$

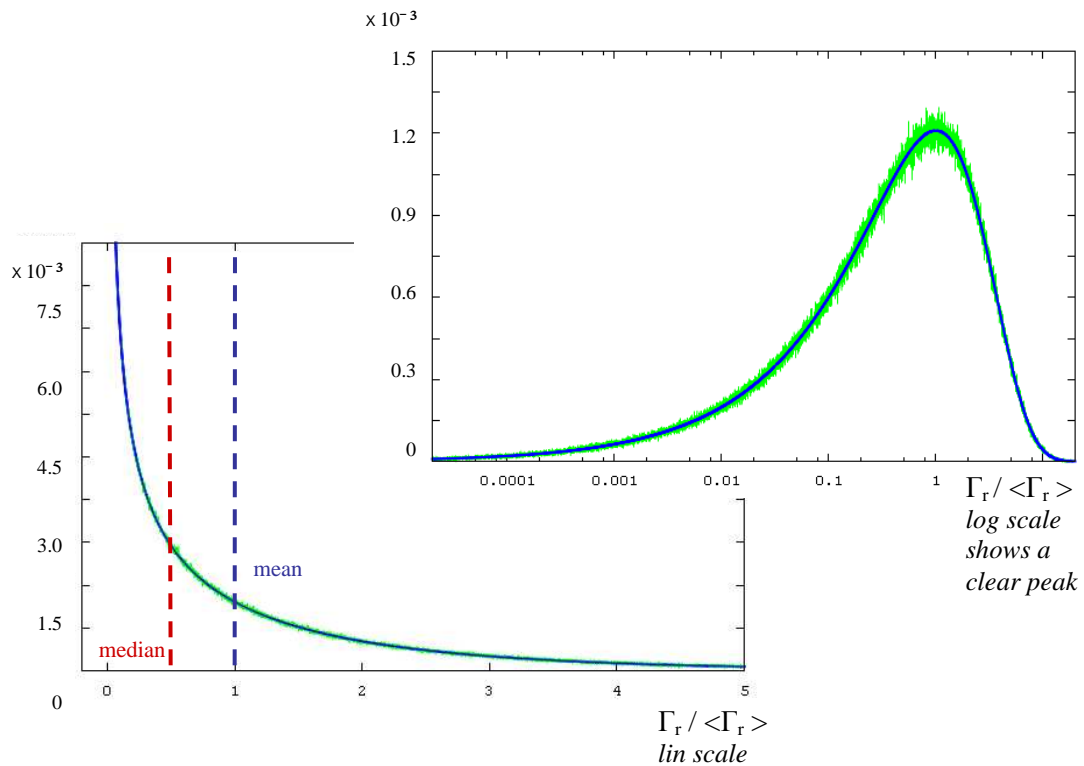
From the fit of this $c(\epsilon)$ to the electron scattering data with the high resolution $\sigma \approx 20 \text{ keV}$

and $\alpha = 5 + 1 = 6$ (as obtained from a MC simulation)

the TU-Darmstadt group has obtained: $D \approx 3 \text{ keV}$ corresponding to $\rho = 333 / \text{MeV}$

3. Porter Thomas fluctuations in n-capture and theory

Determination of missing strength in weak levels is better done after plotting on log-scale for normalized width

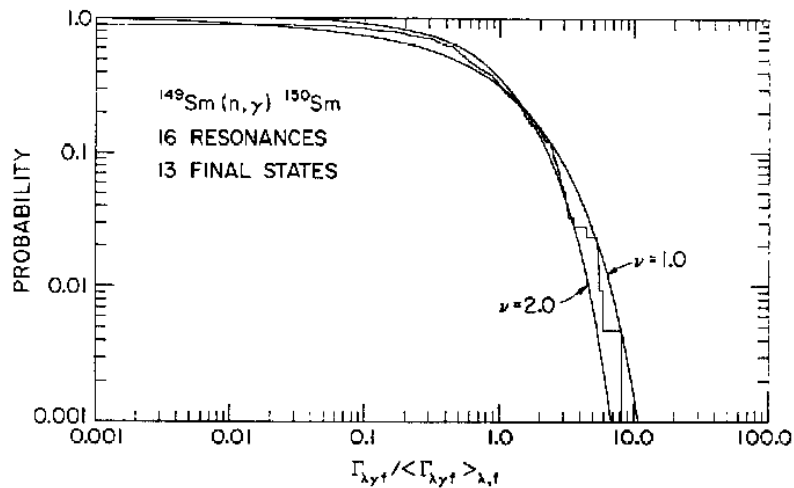


Resonance n-capture by ^{149}Sm :

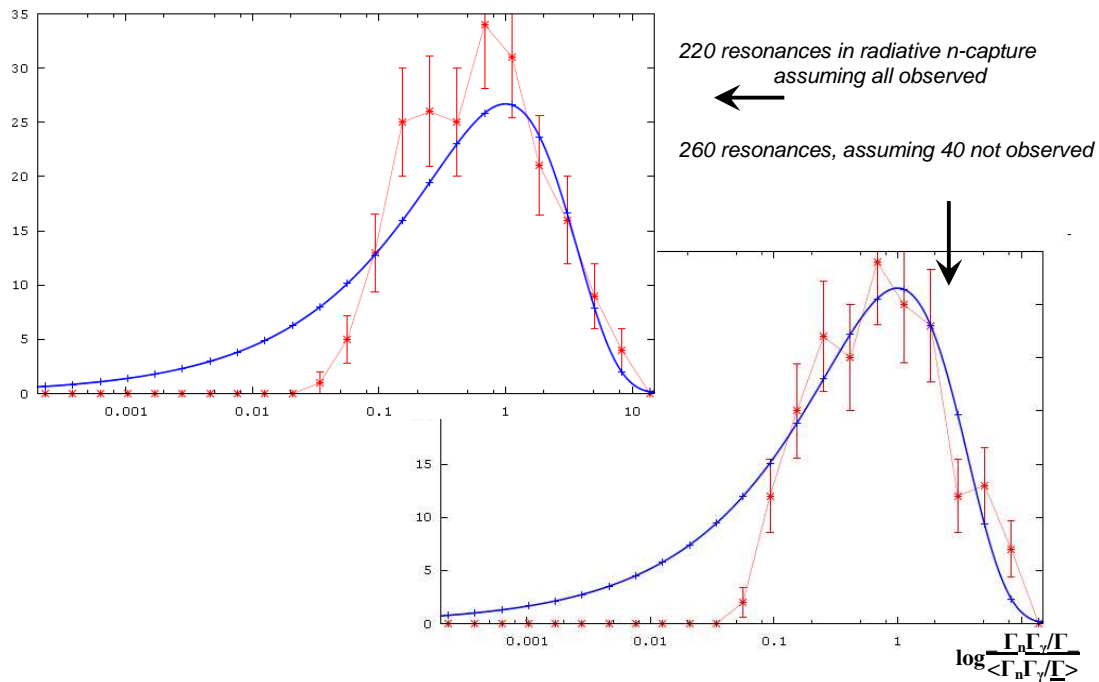
Traditional Porter-Thomas plot does not show a peak;
the derived number of d.o.f. is difficult to interpret.

F. Becvar et al., NPA236 (1974) 173 & NPA236 (1974) 198

Cumulative distribution of the intensities of the transitions from nine resonances with $J^\pi = 3^-$ and seven with $J^\pi = 4^-$ to 13 final levels with $J^\pi = 3^+$ or 4^+ below 2196 keV

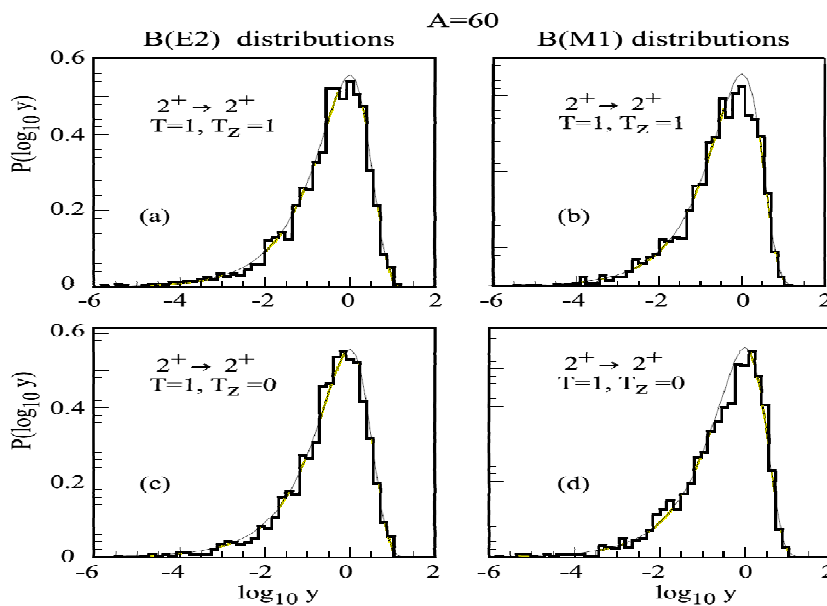


A. Borella et al. PRC 76, 014605 (2007), resonance n-capture on ^{206}Pb at Gelina



Porter-Thomas fluctuations in shell model calculations for $A=60$:

A. Hamoudi, R. Nazmitdinov, E. Shahaliev, and Y. Alhassid, PRC 65 (02) 064311
4290 e.m.matrix elements calculated using the shell model program OXBASH.



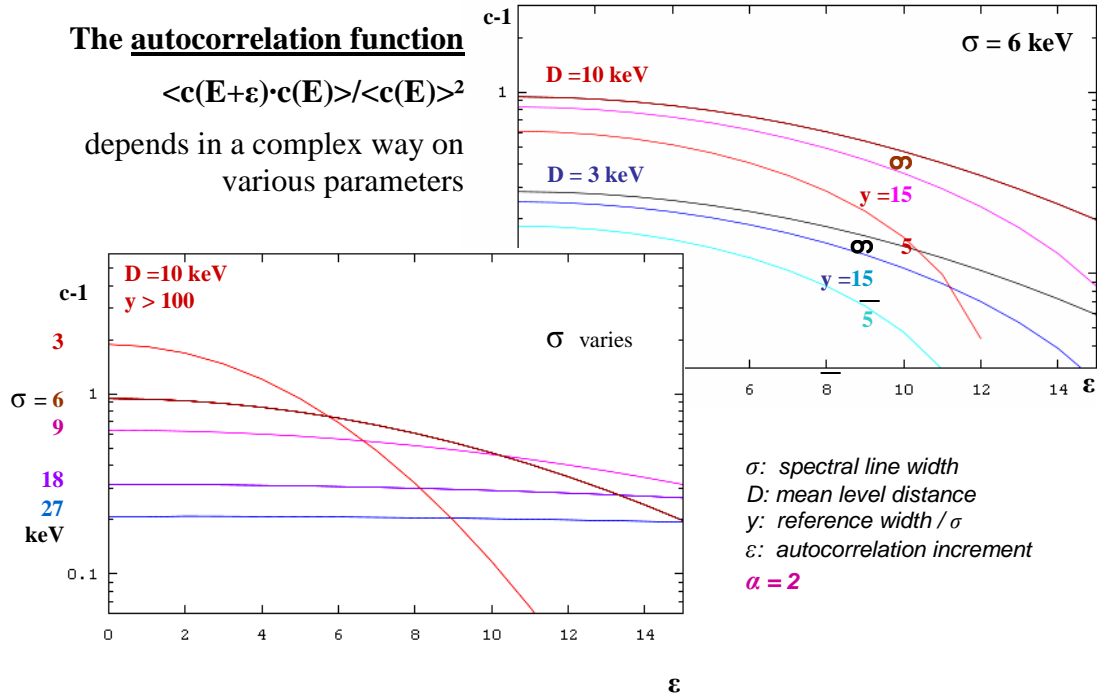
B(E2) (left panels) and B(M1) (right panels) intensity distributions (histograms)
for the $2^+, T = 1 \rightarrow 2^+, T = 1$ transitions in $A = 60$ nuclei:

(a,b) $T_z = 1$; (c,d) $T_z = 0$. The solid lines describe the Porter-Thomas distribution

The autocorrelation function

$$\frac{\langle c(E+\epsilon) \cdot c(E) \rangle}{\langle c(E) \rangle^2}$$

depends in a complex way on various parameters



For the 'quasi-continuum' $\Gamma < D \approx \sigma$

the autocorrelation $c(\epsilon)$ of the spectral strength is given by:

- with Γ : resonance width
- D : mean level distance
- σ : experimental resolution
- α : normalized variance

This equation for $c(\epsilon)$ allows to determine the level distance D and σ from experimental data.

normalized to a Gaussian average $g_{>}(E)$ of width $\sigma_{>} = y \cdot \sigma \gg \sigma$ and their autocovariance $c(\epsilon)-1$ with $c(\epsilon) = \langle d(E) \cdot d(E + \epsilon) \rangle$

Determination of α by Monte-Carlo simulation

of N spectra with resolution σ ($y=\infty$) containing L levels at average distance $D \rightarrow c(0)$

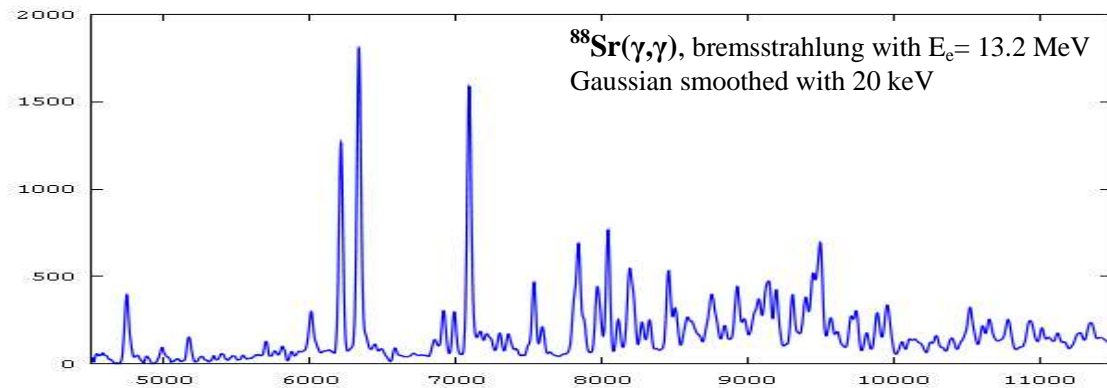
This MC combines 3 level distance distributions with 3 intensity distributions.

For combinations with Porter-Thomas one gets: $2.0 < \alpha < 2.5$.

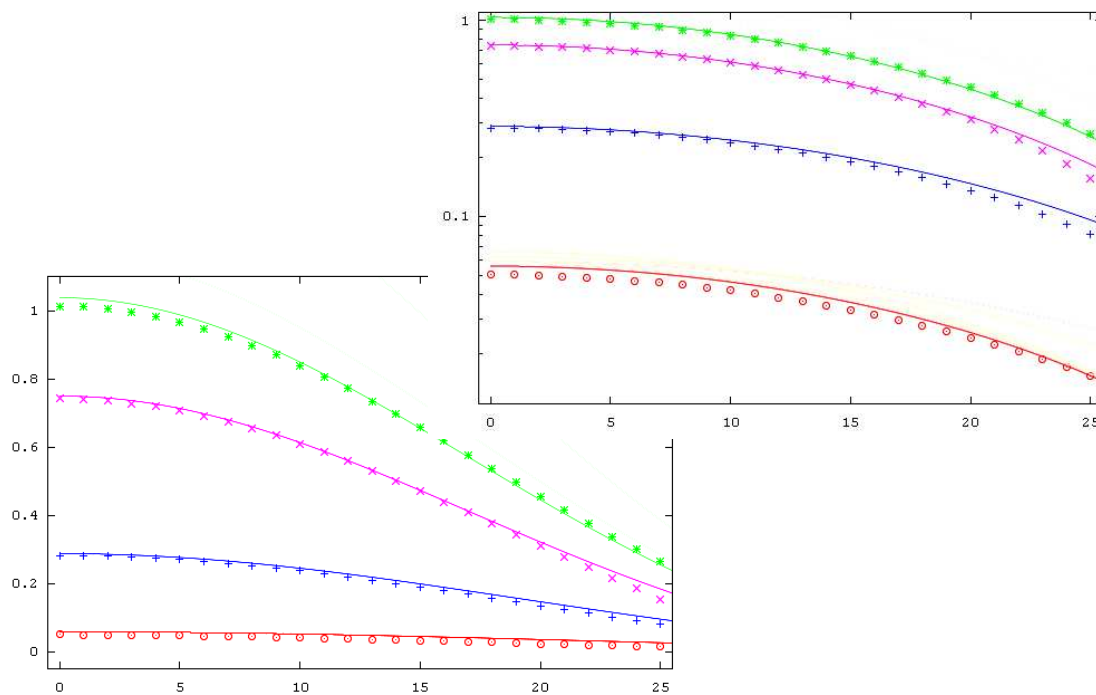
L, D	360, 6	360, 6	430, 5	530, 4	700, 3	430, 5
σ, N	32, 10	32, 100	24, 100	20, 100	15, 100	2, 100
	----- α -----					
Wigner \times PT	1.96	2.04	1.98	2.08	2.03	2.19
\times Gauss	0.97	1.00	1.02	1.07	1.03	1.14
\times rand.	0.42	0.45	0.44	0.46	0.45	0.54
Poisson \times PT	2.28	2.24	2.18	2.24	2.24	2.50
\times Gauss	1.22	1.22	1.21	1.24	1.27	1.42
\times rand.	0.62	0.64	0.63	0.64	0.66	0.82
random \times PT	2.43	2.23	2.24	2.25	2.27	2.50
\times Gauss	1.28	1.19	1.21	1.24	1.24	1.43
\times rand.	0.64	0.64	0.64	0.68	0.67	0.83

4. Discontinuous level density in ^{88}Sr (below S_n) – ELBE data

a) autocorrelation analysis



$^{88}\text{Sr}(\gamma,\gamma)$; the autocorrelation function $\langle c(E+\varepsilon) \cdot c(E) \rangle / \langle c(E) \rangle^2$ is shown in log and lin for 4 regions of photon spectrum: $6 \pm .5$ MeV, $5 \pm .5$ MeV, $8 \pm .5$ MeV, $11 \pm .5$ MeV



From $c(0) - 1$ one can extract $\langle D \rangle = 1/\rho = 2\pi^{1/2} \cdot (c(0) - 1) \cdot \sigma \cdot \alpha^{-1}$;

the variation of $c(\varepsilon)$ with ε allows a check of the experimental resolution σ .

The data show that $D(6 \text{ MeV}) > D(5 \text{ MeV})$ indicating a non-continuous rise of $\rho(E)$.

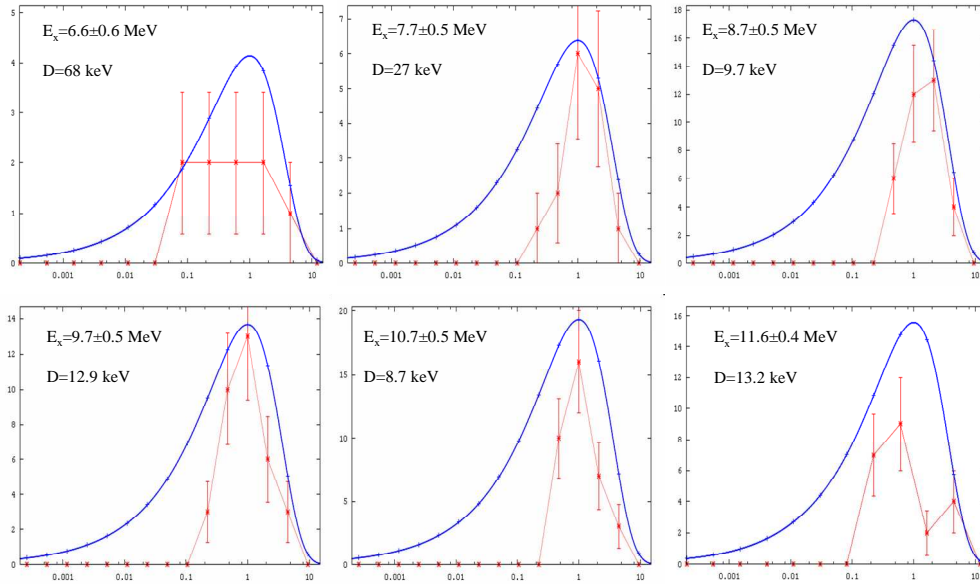
As a test, the results of a ‘traditional’ analysis of the $^{88}\text{Sr}(\gamma,\gamma)$ -spectrum shown above are investigated below with respect to the possible effect of Porter-Thomas fluctuations.

b) Results from the Porter-Thomas analysis
of the list of levels resulting from a conventional analysis.

Also in this analysis, D does not fall continuously with E.

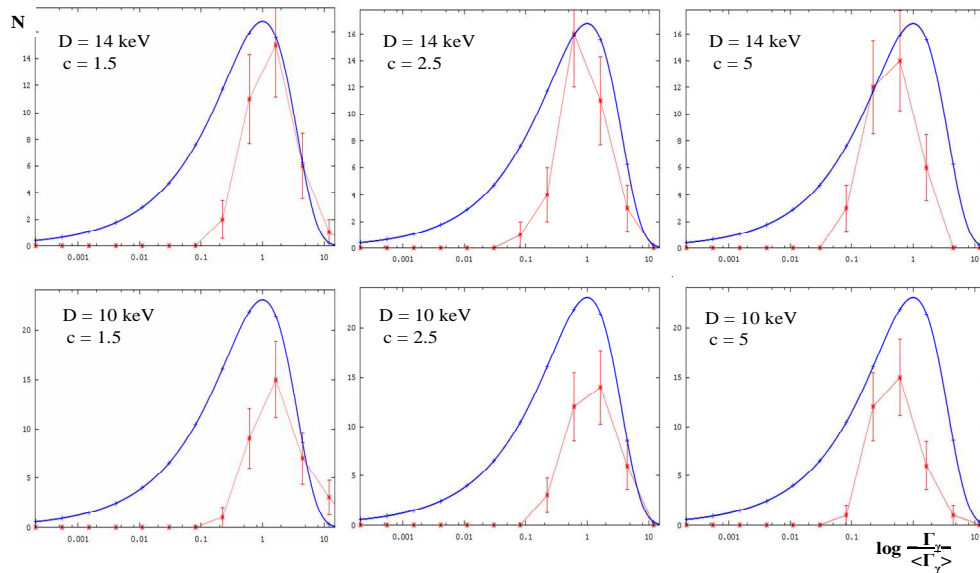
⁸⁸Sr(γ,γ), Porter-Thomas distributions in comparison to data

bremstrahlung with $E_0 = 13.7$ MeV, width distributions for different E_x , total yield determined from spectra.



Sensitivity to level distance D and yield ratio c (total/peaks)

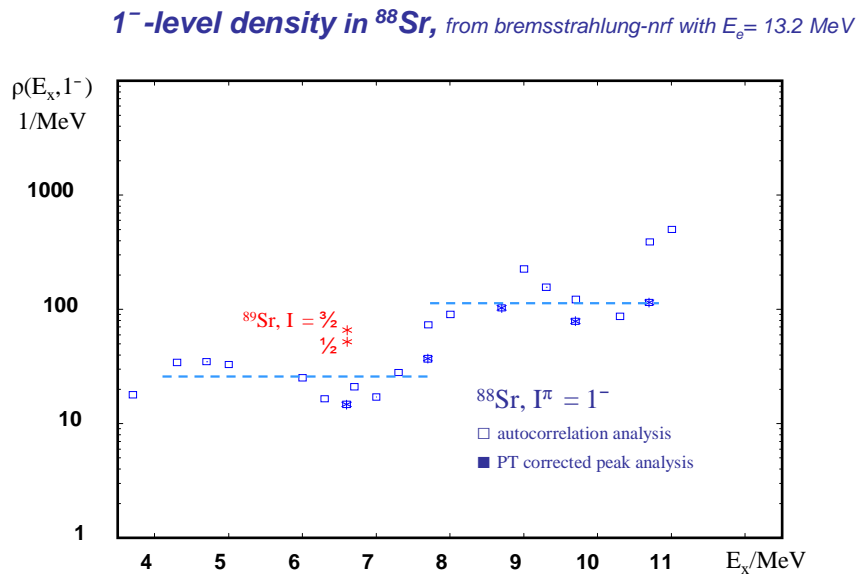
⁸⁸Sr(γ,γ), $E_x = 9.7 \pm 0.5$ MeV



The lower graph shows for the data at $E_x = 9.7 \pm 0.5$ MeV that a small value for D cannot be compensated by stating that more or less intensity is in the analyzed peaks as compared to the quasi-continuum below them ($c >$ or < 2.5).

c) Comparison of the two methods

In this graph, the results of the two types of analysis are compared:
 open squares are from the autocorrelation analysis,
 full dots stem from the Porter-Thomas analysis of the list of levels.



The data are in accord to a stepwise increase of the level density,
 but not to a continuous rise.

Concerning the absolute scale, a reasonable agreement is found to resonance capture
 data for the low spin states in the odd neighbour ⁸⁹Sr, populated in ⁸⁸Sr+n.

Because of its ground state spin n-capture in ⁸⁷Sr leads to high spin levels
 and their density cannot directly be compared to the 1⁻ levels in ⁸⁸Sr.

Non-continuous level densities caused by shell effects
 are under theoretical investigation.

Experimental hints exist for lower excitation energies.